AQA Maths M2

Topic Questions from Papers

Differential Equations

1 A car, of mass 1600 kg, is travelling along a straight horizontal road at a speed of $20\,\text{m}\,\text{s}^{-1}$ when the driving force is removed. The car then freewheels and experiences a resistance force. The resistance force has magnitude $40v$ newtons, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of the car after it has been freewheeling for t seconds.

Find an expression for v in terms of t. *(7 marks)*

(Q5, Jan 2006)

2 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v\,\text{m}\,\text{s}^{-1}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is $25\,\text{m}\,\text{s}^{-1}$.

(a) Show that, at time t

$$v = \left(\frac{20 - t}{4}\right)^2$$

(7 marks)

(b) State the value of t when the particle comes to rest. *(1 mark)*

(Q6, June 2006)

3 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V\,\text{m}\,\text{s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is $60\,\text{m}\,\text{s}^{-1}$.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at $20\,\text{m}\,\text{s}^{-1}$, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v\,\text{m}\,\text{s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -\dfrac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from $20\,\text{m}\,\text{s}^{-1}$ to $10\,\text{m}\,\text{s}^{-1}$. *(6 marks)*

(Q7, Jan 2007)

4 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude λmv, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v$$ *(2 marks)*

(b) The initial speed of the stone is U.

Show that

$$v = U\mathrm{e}^{-\lambda t}$$ *(4 marks)*

(Q7, June 2007)

5 A car of mass 600 kg is driven along a straight horizontal road. The resistance to motion of the car is kv^2 newtons, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the car at time t seconds and k is a constant.

(a) When the engine of the car has power 8 kW, show that the equation of motion of the car is

$$600\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{8000}{v} + kv^2 = 0$$ *(4 marks)*

(b) When the velocity of the car is $20\,\mathrm{m\,s^{-1}}$, the engine is turned off.

(i) Show that the equation of motion of the car now becomes

$$600\frac{\mathrm{d}v}{\mathrm{d}t} = -kv^2$$ *(1 mark)*

(ii) Find, in terms of k, the time taken for the velocity of the car to drop to $10\,\mathrm{m\,s^{-1}}$.
(5 marks)

(Q8, Jan 2008)

6 A car, of mass m, is moving along a straight smooth horizontal road. At time t, the car has speed v. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

$$\frac{dv}{dt} = -0.05v$$ *(1 mark)*

(b) When $t = 0$, the speed of the car is $20\,\mathrm{m\,s^{-1}}$.

Show that $v = 20e^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\mathrm{m\,s^{-1}}$. *(3 marks)*

(Q6, June 2008)

7 A stone, of mass 0.05 kg, is moving along the smooth horizontal floor of a tank, which is filled with oil. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude $0.08v^2$.

(a) Show that

$$\frac{dv}{dt} = -1.6v^2$$ *(2 marks)*

(b) The initial speed of the stone is $3\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \frac{15}{5 + 24t}$$ *(5 marks)*

(Q8, Jan 2009)

8 A stone, of mass m, is moving in a straight line along smooth horizontal ground.

At time t, the stone has speed v. As the stone moves, it experiences a total resistance force of magnitude $\lambda m v^{\frac{3}{2}}$, where λ is a constant. No other horizontal force acts on the stone.

(a) Show that

$$\frac{dv}{dt} = -\lambda v^{\frac{3}{2}}$$ *(2 marks)*

(b) The initial speed of the stone is $9\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \frac{36}{(2 + 3\lambda t)^2}$$ *(7 marks)*

(c) Find, in terms of λ, the time taken for the speed of the stone to drop to $4\,\mathrm{m\,s^{-1}}$. *(3 marks)*

(Q8, June 2009)

9 A golf ball, of mass m kg, is moving in a straight line across smooth horizontal ground. At time t seconds, the golf ball has speed $v\,\mathrm{m\,s^{-1}}$. As the golf ball moves, it experiences a resistance force of magnitude $0.2mv^{\frac{1}{2}}$ newtons until it comes to rest. No other horizontal force acts on the golf ball.

Model the golf ball as a particle.

(a) Show that

$$\frac{dv}{dt} = -0.2v^{\frac{1}{2}}$$ *(1 mark)*

(b) When $t = 0$, the speed of the golf ball is $16\,\mathrm{m\,s^{-1}}$.

Show that $v = (4 - 0.1t)^2$. *(5 marks)*

(c) Find the value of t when $v = 1$. *(3 marks)*

(d) Find the distance travelled by the golf ball as its speed decreases from $16\,\mathrm{m\,s^{-1}}$ to $1\,\mathrm{m\,s^{-1}}$. *(4 marks)*

(Q5, Jan 2010)

10 A particle is moving along a straight line. At time t, the velocity of the particle is v. The acceleration of the particle throughout the motion is $-\dfrac{\lambda}{v^{\frac{1}{4}}}$, where λ is a positive constant. The velocity of the particle is u when $t = 0$.

Find v in terms of u, λ and t. *(7 marks)*

(Q5, June 2010)

11 Vicky has mass 65 kg and is skydiving. She steps out of a helicopter and falls vertically. She then waits a short period of time before opening her parachute. The parachute opens at time $t = 0$ when her speed is $19.6\,\mathrm{m\,s^{-1}}$, and she then experiences an air resistance force of magnitude $260v$ newtons, where $v\,\mathrm{m\,s^{-1}}$ is her speed at time t seconds.

(a) When $t > 0$:

(i) show that the resultant downward force acting on Vicky is

$$65(9.8 - 4v) \text{ newtons}$$ *(1 mark)*

(ii) show that $\dfrac{dv}{dt} = -4(v - 2.45)$. *(2 marks)*

(b) By showing that $\displaystyle\int \frac{1}{v - 2.45}\,dv = -\int 4\,dt$, find v in terms of t. *(5 marks)*

(Q8, Jan 2011)

12 A car, of mass m kg, is moving along a straight horizontal road. At time t seconds, the car has speed $v\,\mathrm{m\,s^{-1}}$. As the car moves, it experiences a resistance force of magnitude $2mv^{\frac{5}{4}}$ newtons. No other horizontal force acts on the car.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -2v^{\frac{5}{4}}$$ *(1 mark)*

(b) The initial speed of the car is $16\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \left(\frac{2}{t+1}\right)^4$$ *(5 marks)*

(Q6, June 2011)

13 Alice places a toy, of mass 0.4 kg, on a slope. The toy is set in motion with an initial velocity of $1\,\mathrm{m\,s^{-1}}$ down the slope. The resultant force acting on the toy is $(2 - 4v)$ newtons, where $v\,\mathrm{m\,s^{-1}}$ is the toy's velocity at time t seconds after it is set in motion.

(a) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -10\,(v - 0.5)$. *(2 marks)*

(b) By using $\displaystyle\int \frac{1}{v - 0.5}\,\mathrm{d}v = -\int 10\,\mathrm{d}t$, find v in terms of t. *(5 marks)*

(c) Find the time taken for the toy's velocity to reduce to $0.55\,\mathrm{m\,s^{-1}}$. *(3 marks)*

(Q6, Jan 2012)

14 A stone, of mass 5 kg, is projected vertically downwards, in a viscous liquid, with an initial speed of $7\,\mathrm{m\,s^{-1}}$.

At time t seconds after it is projected, the stone has speed $v\,\mathrm{m\,s^{-1}}$ and it experiences a resistance force of magnitude $9.8v$ newtons.

(a) When $t \geqslant 0$, show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -1.96(v - 5)$$ *(2 marks)*

(b) Find v in terms of t. *(5 marks)*

(Q7, June 2012)

15 A particle, of mass 12 kg, is moving along a straight horizontal line. At time t seconds, the particle has speed $v\,\mathrm{m\,s^{-1}}$. As the particle moves, it experiences a resistance force of magnitude $4v^{\frac{1}{3}}$. No other horizontal force acts on the particle.

The initial speed of the particle is $8\,\mathrm{m\,s^{-1}}$.

(a) Show that

$$v = \left(4 - \frac{2}{9}t\right)^{\frac{3}{2}}$$

(6 marks)

(b) Find the value of t when the particle comes to rest. *(1 mark)*

(Q5, Jan 2013)

16 A car accelerates from rest along a straight horizontal road.

The car's engine produces a constant horizontal force of magnitude 4000 N.

At time t seconds, the speed of the car is $v\,\mathrm{m\,s^{-1}}$, and a resistance force of magnitude $40v$ newtons acts upon the car.

The mass of the car is 1600 kg.

(a) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{100 - v}{40}$. *(2 marks)*

(b) Find the velocity of the car at time t. *(6 marks)*

(Q6, June 2013)